

My Mathematical Life

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Many mathematical biographies leave the reader with the impression that the subject's mathematical life was almost inevitable at least from an early stage in their life. Mine is going to be different. An academic career seemed very remote at many stages of my life and even if it did look possible, mathematics was by no means the only possibility and even after I started working on a PhD my eventual direction of research was by no means obvious, indeed it might be viewed as a matter of chance.

My father, Ernest (Ernie), was born in the early nineteen twenties and his father, who was in a second marriage, died of old age when Ernie was 3 months old, leaving my grandmother a single mother with three children to raise in the town of Battle, near Hastings. Ernie left school at the age of 14 and spent a year assisting a milkman before becoming apprenticed to a market gardener. At the start of the second world war, he enlisted in the RAF and was trained as a radio operator. As he put it, he spent the war operating a radio in a hut on Dartmoor, which is how he met my mother, Lilian (Lil). She also came from a relatively impoverished background in Plymouth. Growing up, her family lived in what is now a tourist attraction in Plymouth called the Elizabethan House, which may sound quite affluent, but the reality is that the family of two adults and three children lived in one room of the house and shared a bathroom with the occupants of the other rooms. When the war started, Lil got a job with the NAAFI (Navy Army and Air Force Institutes) which ran shops, canteens etc for the armed forces. This was where she met Ernie. At the end of the war, Ernie spent a year with the Military Police in Germany before returning to Plymouth, marrying Lil and, after his experience with the Military Police, joining the City of Plymouth police force.

I was born in December 1947. At that time, we rented a flat above a pawn shop in Union Street, a street mainly known for providing recreational activities for sailors, Plymouth being a major military port. Although the flat was quite a decent size, it did not run to such luxuries as an inside toilet—that was at the end of the garden! In due course, I attended the local primary school. I guess there were some signs of my mathematical ability there. I remember Friday afternoon mental arithmetic challenges which I did well at. Handwriting was quite another story! At that time, progression to secondary schools was determined by the results of

an exam called the “Eleven plus”, although in Plymouth one could take it at age 10 and again at 11. My school was not one where they expected pupils to progress to grammar school afterwards, but in my year three of us did.

I started at Devonport High School for Boys in the autumn of 1958. Given my background, it seemed a very luxurious setting, even having inside toilets which my primary school did not have! A year later my family obtained a council house (built by and rented out by the local authority) and I finally got to experience the luxury of indoor toilets at home. At the start of the second year there I made a friendship which influenced my life substantially over the next decade, namely with Richard Haydon (Yes! the mathematician.) I was already an avid reader of Science Fiction, mainly from the local public library, but he introduced me to some of the classical writers by lending me copies of, for example, *The Foundation Trilogy* and *The Space Merchants*. His parents inadvertently caused me great embarrassment the first time that I ate a meal in their house as I had never before had to deal with multiple knives and forks and had no idea which to use first! They more than made up for this in later years when they gave Richard and myself Bridge practice sessions—they played as a partnership for the county team.

My parents encouraged reading as much as they could, but the local library had been of great importance in providing reading material in my early years. Our move of house triggered a major improvement there. Previously, my local library was a small one room affair nearby. In the new location the nearest public library was a fair distance away but was much better stocked than the previous one. A bonus was that the journey to and from school involved two bus journeys, with one of the possible transfer points being directly opposite the main city public library. The school also had what, to me at least, seemed a very well-equipped library as well. At this distance in time, I can only remember two borrowings precisely. They were *Das Kapital*, which seemed very risqué to me at the time, and Newman's *The World of Mathematics* which opened my eyes to the vast scope of mathematics beyond the school curriculum.

My interests at school were predominantly in the sciences, Physics and Chemistry, until I was around fourteen. By then I realised the importance of mathematics for any scientific career, began to take more interest in it and thoroughly enjoyed it. The head of Mathematics in the school at the time was Eric Warne. At the start of sixth form he took Richard and myself into his store room, picked out about two dozen books for each of us and said "That's your next two years' work, get on with it." We both obtained Scholarships to Cambridge colleges, which were in addition to the (then) normal grants (not loans!) In the end I was also given a scholarship from International Computers and Tabulators (ICT) which replaced the normal grant. Obtaining these scholarships involved

staying for an extra term in a third year in the sixth form, but the school had nothing more to offer beyond the end of 1965. ICT offered me (paid) work experience as a computer operator, which I thoroughly enjoyed. As you will see later, it came in useful in the first year of my PhD.

I can't really say that I enjoyed my time at Cambridge---I just didn't really fit in. After a year, I took a year's leave of absence. A year of "real" work of various kinds persuaded me to go back. I managed a reasonable degree, but not a great one. Not good enough to get a grant to study for a PhD anyway. By chance, I managed to get accepted to study for a PhD at Chelsea College, London, but without any funding. My father paid the first year's fees and I looked for a job to fund my living expenses. This was in the run up to decimalization of the UK currency so there was a lot of work in financial computer programming to be done. On the basis of my experience at ICT, I got a job with an insurance company (NMLAS) which wanted to increase the number of hours their computer was operated each day, so were introducing shift work for their computer operators. One of the existing operators was a young woman whose mother was not happy with her travelling late at night, so we came to an arrangement where she worked permanent day shifts and I did permanent evening shifts. That meant that I could put in all the expected presence in college as well as holding down a full-time job, which actually involved a lot of time just waiting for programs to finish, during which time I could study. The only possible temptation to give up mathematics came on my last day at NMLAS when the chief actuary asked to see me. He explained that in London it is the stockbrokers who get to be Lord Mayor etc, "but we are the ones who make the real money", and "are you interested?" I wasn't.

My supervisor in Chelsea was Francis (Frank) Jellett, who set me off studying compact convex sets. He must have been satisfied with my progress during the first year as he arranged for me to get a college studentship, covering fees and a maintenance grant for the next two years. Early in the second year of my study I solved a problem that he had posed (although I later found out that the problem had originally been slightly different!) and had a paper accepted in the *Quarterly Journal of Mathematics*. It seemed to me that not a lot was happening in the compact convex set theory world then after many years of progress by Alfsen's group, apart from applications to operator algebras, where I had no background. Around the same time, Frank got me to read A.J. Ellis's PhD thesis in order to see what a thesis should look like. It was on the duality of ordered Banach spaces and it struck me that there were a lot of outstanding questions about operators between ordered Banach spaces. I managed to answer several of these in a paper that was published in the *Proceedings of the London Mathematical Society*. Frank had got me into the good habit of writing up any results that I obtained properly, so between the two accepted papers, and a few other minor results all I needed for a

thesis was an introductory chapter. I took a week's holiday in the summer of 1972, spent two weeks hand writing the thesis then bought a second-hand typewriter and learned the art of leaving the appropriate size gaps for inserting mathematical symbols by hand. (How I wish that I had access to LaTeX then!)

At the start of the 1972-1973 academic year a Turkish student, Şafak Alpay came to Chelsea to study for a PhD under Prof. J.L.B. Cooper, the head of department, but quite soon transferred to Frank Jellett as supervisor. Although our contact during the remainder of my time in Chelsea was rather limited, this connection was to have great influence on my mathematical career in later years.

It wasn't until the end of March 1973 that my PhD viva took place. After writing up the thesis I again felt the need to query where I could go next. As many people realize now, ordered Banach spaces don't have a lot of structure to work with. In 1973, I came across several publications that were eventually to prove pivotal in setting the direction of my future work. These were Fremlin's series of papers on *Abstract Köthe Spaces* and the collected papers of Nakano. By August of 1973, I had submitted a further paper to *Mathematische Annalen* and really had to think hard about getting a job.

I had applied to some lectureships without success, although there weren't many around at the time. A contact at Chelsea informed me of a post at the University of Ife in Nigeria. The vice-Chancellor was in London at the time, so the application process was straightforward, involving only an interview with him. I was probably the only applicant, as I learnt of my success quite quickly. My last couple of months in London went by in rather a blur. I had to learn to drive, before which I needed to get spectacles, get a passport, have several vaccinations and obtain various medications. Given that I had never flown before, and never left Great Britain, I suspect that the lack of time to think about exactly what I was letting myself in for was probably a good thing.

So, in the autumn of 1973, I found myself on a Boeing 707 headed from Heathrow to Lagos. Five of the passengers on board transferred to a DC3 flight from Lagos to Ibadan. Three of the other passengers were Bob and Angela Welch and their daughter Rachel. Bob was taking up a post in the English Department at Ife and the university had sent a car to meet the four of us. The fifth passenger was an existing member of staff in Ife who, in what I soon learned was typical Nigerian behaviour, persuaded the car driver to include him on the journey. The Welch's and I ended up living in the same housing estate between the university campus and the town of Ile-Ife itself. They were good friends during the year that I spend there and we kept in contact for many years afterwards.

Life in Nigeria was difficult in many ways at the time. Economic problems, and sheer incompetence, caused various commodities to sporadically be in short supply. One incident I remember was when the country's only oil refinery was closed down for maintenance and no-one had thought of applying for an import permit for refined oil products, so the country had no petrol for a couple of weeks. There was a general aura of corruption and violence just under the surface, but on the other hand, apart from the few who were exiled, leaders of the Biafran rebellion were treated remarkably well. I knew an ex-Biafran fighter jet pilot who was studying at the university with the help of a government grant. The same economic problems meant that subscriptions to mathematical journals were erratic and late. The year that I spent there was, though, a great experience that I am, in retrospect, glad that I had. But after one academic year the difficulties in keeping up research activity plus my dissatisfaction with the department in general left me open to other offers.

In my final year in Chelsea Finbarr Holland, from University College Cork, was spending the year on sabbatical at Chelsea. The department held open drop-in tutorial sessions on Friday afternoons where students could come along and seek help with any mathematics course from any level. These were staffed by several members of academic staff and research students. Finbarr and myself were two of the team during at least one term. The sessions weren't all that busy so we were able to discuss mathematics and he got to see me teaching. For the 1974–76 academic years, Prof. Patrick (Paddy) Barry, the head of department in Cork, was appointed to be college Vice-President—a full time position. Finbarr became acting head of department and needed to find, at short notice, a temporary replacement for Paddy. He wrote to me offering me the position (I'm sure nowhere could do such a thing nowadays!) and I jumped at the chance.

After a year in Nigeria, Cork felt like a return to civilization. Everyone, mathematics staff, other university staff and people in general were very friendly. I will mention only the three mathematicians there with whom I collaborated either in that year or soon after. I shared an office with A.K. (Tony) Seda, with a remarkably friendly relationship. As a result of an afternoon spent briefing each other on our own research we wrote a short paper for *Annales de la Société Scientifique de Bruxelles*. R.E. (Robin) Harte was closest to my research area in the department at the time. We spent quite a lot of time discussing mathematics over the year, with no precise outcome at the time, but several joint papers resulted a few years later. J.J.M. (Jim) Chadwick had been in Cork but by then was at Trinity College Dublin (TCD), though still a frequent visitor to Cork. Again, relatively close research interests brought us together and we eventually published a joint paper in the *Bulletin of the London Mathematical Society*. These were my first jointly authored papers, a phenomenon that is much more common

now than it was in those days. Apart from this, the year was very productive in research output but with most publications only appearing after I left Cork.

Everyone in Cork was aware that I needed to be on the look out for a permanent position and someone (I don't remember who) informed me near the end of my first year in Cork of a vacancy at Queen's University Belfast (QUB), which I duly applied for. This was again a post to be filled at short notice. In this case the post was to start as early as possible in October. I don't remember the exact dates now, but there was precisely one week between the interviews for the position and my start date. I ended up being successful and then had to go back to Cork and see how people reacted to my early departure. As it turned out everyone involved (Finbarr, the college President and my landlady) went out of their way to be of assistance and there was no difficulty.

Apart from ongoing research into vector lattices, the most significant activity of my first year at QUB came about after I received a letter from Şafak Alpay informing me of the *International Congress of Functional Analysis* to be held in Silivri, Turkey, in September 1976. This was exceptionally well funded with everything paid for from air fares, accommodation, meals and even up to alcohol! This was my first experience of a specialist mathematical conference and I (over) prepared a talk on the structure space and ideal centre of Banach lattices. But the most important outcome, in the short term, involved upper and lower Fredholm spectra. Robin Harte gave a talk on this explaining work that he and J.J. Buoni had done. The main result involved three conditions, (A), (B) and (C). They had proved that (A) implies (B) and that (B) implies (C) but could not show that (C) implies (A). Robin's talk was last in the afternoon with a break then before the evening meal. I managed to fill in the last step before the meal and stopped at Robin's room to give him the proof before the meal. It turned out that someone else also completed this step but, not knowing Robin personally, waited until the meal time before letting him know. On this basis, Robin gave me priority and my name ended up on the paper. Robin later joked that it said a lot about North-South Irish relationships that collaboration between Belfast and Cork had to take place via Turkey! Robin and I published two more papers on the same topic and Jim Chadwick and myself published a slight variant of the original construction.

Later in 1976 I received a letter from Richard Haydon, who had been at an Oberwolfach workshop on Banach lattices. He mentioned a problem posed by A. Goulet de Rugy asking if there was a Banach lattice with trivial centre. Given that he had identified the centre with a space of continuous real valued functions on a compact (not necessarily Hausdorff) topological space and had also shown that in such spaces it was possible that no two non-empty open sets were disjoint, it seemed to me obvious that he had already solved the problem. Or was I missing something? I wrote to him pointing this out and he wrote back quickly agreeing.

It gave me much more confidence in my own work if someone of his stature could miss such a point!

More importantly, in the long run, was learning of the existence of the Oberwolfach workshops. I managed to get onto the list of invitees and I think I was at all the rest of them until they died out. Almost all my contacts with what is now referred to as the positivity community came about via Oberwolfach. Undoubtedly the most significant was my meeting with Yuri Abramovich. He had finally managed to leave the Soviet Union and had unexpectedly managed to get to Oberwolfach during one of the workshops. I knew of his work, although we had not been in touch. Surprisingly, he also knew of my work. We had a very productive talk and laid the foundations for a couple of NATO collaborative research awards. I had two very productive visits to him in Indianapolis, which gave me the opportunity to spend time with C.D. (Roko) Aliprantis and Owen Burkinshaw, and he also visited Belfast. Those visits made me greatly envious of an environment that contained several people who could work together. I have never been in that situation and still regret that lack and wonder what I might have done in such a group. Even at that time, collaborating by email still involved clashes between the use of some symbols in emails and in LaTeX. Apart from our research together, Yuri played a very important role in the promotion of the subject area by convincing Kluwer that a specialist journal was worth considering. Then he persuaded me to get involved. I carried out the market research that finally convinced Kluwer that the project was feasible, recruited an initial board of editors and (in the absence of any competition) designed the journal logo (although Kluwer's design department must take the blame for it being upside down!)

I was lucky enough to supervise some outstanding PhD students at QUB, even though the numbers were not huge. The first was Peter McPolin, who started his studies with me after completing his BSc in QUB in 1979. He completed an MSc thesis at the end of the first year, as was then the custom, then started working on his PhD. At the end of the three years, which was the limit of his grant, he was offered and accepted a position at St. Mary's Teacher Training College, Belfast (as it was then called) where he had an extremely successful career until his unfortunately early demise on 1st March 2020. Starting at St. Mary's delayed the submission of his thesis by a year, but the wait was worth it as he used the time to polish and fine tune the thesis. Although he published no more mathematical papers after the three that resulted from his thesis work, he undoubtedly played an important role in raising the standard of mathematics teaching in Northern Ireland.

Late in the 1980's Şafak Alpay contacted me concerning an M.Sc. student of his at METU who wanted to study for a PhD and, most importantly, had funding to

study abroad. That was Zafer Ercan who studied with me until he completed his PhD in 1993. It was an extremely sound thesis and three publications resulted from it. He returned to Turkey on completion of his thesis and held several university positions afterwards. Apart from 66 mathematical publications listed by MathSciNet, he helped boost the subject area of Positivity in Turkey in many ways, including successfully supervising two PhD students.

In 1993, I was contacted by a young Chinese mathematician called Chen Zili who wanted to study for a PhD. He had a few reasonable looking papers published in Chinese journals but no source of funding. At the time QUB had a few studentships available for international students and I was lucky enough to be able to obtain one of these for him. He started working with me in 1994 and completed an outstanding thesis in August 1997. No less than eight publications resulted either directly or indirectly from his thesis. On completion he returned to Chengdu where he remained until his retirement. MathSciNet lists 63 publications under his name.

In 1998 Michael Elliott completed his B.Sc. at QUB and started studying with me. He submitted a thesis in 2001 after a period of study that was marked only by a few personal eccentricities. He has never sought an academic position since then nor shown any interest in publishing work from his thesis, even though it contains many new results. The only paper that he has been induced to publish in the field solved the longstanding problem of whether or not operator lattice operations had to satisfy the Riesz-Kantorovich formulae. He has made significant (if not complete) progress on two other problems of comparable renown. It is a major regret of mine that I have been unable to persuade him to publish more!

My final student, Stuart McKenzie, had no interest in positivity nor in working with me! He started working with someone else in another mathematical area in 2010. After a year it had become clear that there was a complete mismatch between the supervisor's expectations and his own, at least as far as hours of work were concerned. Stuart made it clear then that he was aiming at obtaining a PhD rather than entering a research career. I was the only person in the department willing to take him on with the mathematical background that he had, so he started working with me in 2011. The first year went reasonably well, but in September 2012 I was diagnosed with a duodenal tumour which required immediate major surgery, followed by chemotherapy. As a result, I was unable to return to work until the summer of 2013. Michael Elliott, at the time, had a temporary part-time teaching position with us and agreed to take over Stuart's supervision until I returned. It is to the credit of both Michael and Stuart that Stuart successfully completed his Ph.D. in November 2014.

Of course, my employment in QUB involved much more than undertaking research. Apart from the ongoing commitment to teaching, I had various administrative tasks to perform. When I came to QUB in 1975 the Department of Pure Mathematics was a unit in the Faculty of Science on an administrative level, though not a staffing or funding level, with departments like Physics. There was, though, an informal grouping of Pure Mathematics, Applied Mathematics & Theoretical Physics, Physics and Computer Science. Heads of department were appointed for life.

In 1985 I was promoted to Reader. In 1989 the university embarked on a major reorganisation that, in our case merged the two mathematical departments and Physics, with Computer Science escaping from the Faculty completely. The new School, as it was termed, was dominated both financially and in staff numbers by Physics. A rather arcane administrative structure was adopted which saw the school divided into four undergraduate teaching areas (Pure Mathematics, Applied Mathematics, Statistics & Operational Research and Physics) and several research areas (Pure Mathematics, Theoretical Physics, Statistics & OR and several branches of experimental physics.) All academic staff were allocated to one teaching and one research area and each teaching or research area had its own head. In Pure Mathematics everyone was, of course, in both the teaching and research areas so we had two “heads” in charge of exactly the same group of people. It was actually more complicated than this, but you really don't want to know any more! Suggesting that there should be a common head was vetoed by the physicists of course! I ended up as Head of Undergraduate teaching in Pure Mathematics for 1989-1992 and 1998-2004 and Head of Research for 1992-1995 and 2005-2011.

In the early 1980's we became aware of the existence of these things called “microcomputers”. The (then permanent) head of department bought two ITT2020's (a variant of the Apple II) and, knowing that I had some previous exposure to computers, asked me to investigate what could be done with them. I developed some software for teaching purposes though becoming more convinced that the future lay in teaching students what computers could do for mathematicians, rather than using computers to teach mathematics to students. Nevertheless, it was the right time to get involved in such matters and we dabbled in teaching teachers about the topic. Having charge, in practise, of our computers and equipment budget meant that I could, without formal authority, steer our computing direction. Thus, a few years later when we had IBM PC compatible hardware, I was able to get TeX installed and train secretarial staff in its use, as long as the head of department got to have the latest toy to play with!

In the mid 1980's I was getting really disheartened with the state of Pure Mathematics in QUB, and started thinking about alternatives, including switching

subject area to Computer Science. I got as far as talking about this to the head of Computer Science, but things started improving in Pure Mathematics before I committed myself.

Around 1987, whilst at a meeting of a subcommittee of the Royal Irish Academy in Dublin, I heard people talking about a program called *Mathematica*. It sounded interesting enough that I read up a bit about it and about computer algebra in general. A few months later, the computing committee of the old informal school was discussing the design of a new MSc degree on Computing for Scientists. I mentioned computer algebra as being something worth investigating, with the hope that I might be able to sit in on lectures on the topic. The chair of the committee was very enthusiastic, and immediately put me down to teach it! The only software then available in the university was *REDUCE* which ran on the two mainframe computers that the university then had. The course seemed to go down well, the only problem was that with six students using *REDUCE* at one time, the mainframe computer we used was running near capacity and all other users had to be thrown off for the duration of the exam!

By 1991, we were in a position to put on an undergraduate course on *Mathematica* using in-house PCs. This gained numbers rapidly, eventually attracting students from Computer Science, Electrical Engineering, Finance and the two Belfast Teacher Training Colleges. Before physicists in QUB really became involved with it, I was able to contribute *Mathematica* expertise to publications in some areas of physics. As a result, for several years I ran introductory courses on *Mathematica* for postgraduate physics students and later for the university's postgraduate training centre.

Over the years, I became involved, either as an individual or in some administrative capacity, with the development of several other courses that were, for the time, innovative in some way. In 1980, the UK published the Cockcroft Report on the teaching of mathematics in school. At the suggestion of Mike Holcombe, we set up a course on *Mathematical Investigations*, with emphasis on solving problems using mathematics that students already knew rather than teaching them more mathematics. Initially this was aimed at potential school teachers but evolved considerably in later years. One of the perennial problems that almost all mathematics departments have is provision of basic, almost remedial, mathematics for a broad cohort of students. We had considerable success with a computer-based approach using *CALMAT* for several years. The most distinctive development that worked well for quite a few years started when I attended a workshop in Cambridge on teaching mathematics for access students. The very first speaker started by saying “of course you can't provide an access route to a mathematics degree.” I spent the rest of the day working out how and why QUB, unlike universities in England and Wales, could do. We already had a

Level 0 year for entrants with A-levels that weren't quite good enough for normal level 1 entry. Combining Access courses in regional colleges that taught study skills as well as some mathematics with entry into level 0, ought to work. After some persuasion within the university, we approached potential providers of Access courses, many of whom were very keen on the idea. The scheme ran for several years, until QUB abolished level 0! We found a dichotomy amongst students on the scheme. Either they dropped out early for financial reasons, or else did very well obtaining at least upper second class honours. There were certainly capable young people out there who were getting failed by the normal educational route.

By the mid 1990's I was completely involved with the running of Pure Mathematics in some form or other and that left only time for my day to day teaching and my research, which is all either uninteresting or a matter of public record. None of these topics merits being described further here. In the early part of the 21st century the UK abolished compulsory retirement at the age of 65, so I was able to continue working until my 68th birthday, at which time a voluntary severance scheme tempted me. Since then, my research has slowed down, especially very recently, but not stopped. I feel deeply appreciative that people still value what little I have been able to contribute to the area of positivity.

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